Parallel and Multilevel Algorithms for Computational Partial Differential Equations

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ABSTRACT

The efficient and reliable solution of partial differential equations (PDEs) plays an essential role in a very large number of applications in business, engineering and science, ranging from the modelling of financial markets through to the prediction of complex fluid flows. This paper presents a discussion of alternative approaches to the fast solution of elliptic and parabolic PDEs based upon the use of parallel, adaptive and multilevel algorithms. Mesh adaptivity is essential to ensure that the solution is approximated to different local resolutions across the domain according to its local properties, whilst the multilevel algorithms ensure that the computational time to solve the resulting finite element equations is proportional to the number of unknowns. Applying these techniques efficiently on parallel computer architectures leads to significant practical problems. Difficulties addressed in this paper include how to handle the coarse grid operations efficiently in parallel and the dynamic load-balancing problem that arises when the finite element mesh is adapted.

Keywords: Partial Differential Equations, Parallel Computing, Multilevel Algorithms, Adaptive Mesh Refinement.

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